# TRANSFER OF ENERGY FROM AN EVAPORATING DROP IN A VAPOR MEDIUM

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The article considers the temperature distribution around an evaporating drop in a vapor medium. The transfer of energy is effected by molecular thermal conductivity, convection, and radiation. The mean length of the free flight path of the radiation considerably exceeds the characteristic distance at which the temperature changes. The times required for relaxation of the temperature to a steady-state value are determined, as well as the characteristic distances at which the temperature distribution changes.

1. Basic Equations. As is well known, the transfer of energy by convection, molecular thermal conductivity, and radiation is described by the equations [1, 2]

$$\rho c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}\right)=\left(\frac{\partial}{\partial r}+\frac{2}{r}\right)\left(\varkappa\frac{\partial T}{\partial r}-S\right) \tag{1.1}$$

$$\left(\frac{\partial}{\partial r}+\frac{2}{r}\right)S=\alpha c(U_p-U),\quad v=\frac{\rho_a v_a a^2}{\rho r^2},\quad U_p=\frac{4\sigma}{c}T^4$$

Here r is the distance to the center of the drop; t is the time; $\rho$, $c_p$, v, are the density, the heat capacity, and the radial component of the velocity of the vapor medium; $a$ is the radius of the drop; the subscript $a$ denotes quantities referring to the surface of the drop; T, $\varkappa$ are the temperature and the molecular thermal conductivity; $1/\alpha$ is the mean length of the free flight path of the radiation; $\sigma$ is the Stefan–Boltzmann constant; c is the speed of light. The last equation is justified with local thermodynamic equilibrium, for which [3] contains an investigation of the conditions for its applicability.

The density of the radiant energy U and the radial component of the density of the radiant heat flux S are connected with the intensity I(r, $\theta$) in the following manner:

$$U=\frac{2\pi}{c}\int_0^\pi I\sin\theta\,d\theta,\qquad S=2\pi\int_0^\pi I\cos\theta\sin\theta\,d\theta$$

The intensity of the radiation far from the drop $I_\infty=(\sigma/\pi)\,T_\infty^4$ is determined by the temperature of the medium, i.e., $T_\infty$.

It is assumed that the characteristic distance $r_0$, at which the temperature $T_\infty$ is established, satisfies the condition $\alpha r_0 \ll 1$. In this case, with an accuracy up to terms on the order of $\alpha r_0$, the intensity of the radiation is equal to

$$I(r,\theta)=I(a,\theta)\quad \text{at}\quad \theta\leqslant \arcsin a/r$$

$$I(r,\theta)=\frac{\sigma}{\pi}T_\infty^4\quad \text{at}\ \arcsin\frac{a}{r}<\theta\leqslant\pi$$

$$I(a,\theta)=\varepsilon(\sigma/\pi)T_a^4+(1-\varepsilon)(\sigma/\pi)T_\infty^4$$

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Here $\varepsilon$ is the effective degree of blackness. It follows from this that

$$U = \frac{4\sigma}{c} T_\infty^4 + \frac{2\varepsilon\sigma}{c}(T_a^4 - T_\infty^4)\left[1 - \left(1 - \frac{a^2}{r^2}\right)^{1/2}\right] \tag{1.2}$$

From (1.1) and (1.2) we can obtain an equation describing the temperature distribution around a drop,

$$\rho c_p \frac{\partial T}{\partial t} + \rho_a c_p v_a \frac{a^2}{r^2}\frac{\partial T}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \varkappa \frac{\partial T}{\partial r} - 4\alpha\sigma(T^4 - T_\infty^4) + 2\varepsilon\alpha\sigma(T_a^4 - T_\infty^4)\left[1 - \left(1 - \frac{a^2}{r^2}\right)^{1/2}\right] \tag{1.3}$$

The initial and boundary conditions have the form

$$T(r, 0) = T_\infty, \quad r > a; \quad T(a, t) = T_a, \quad t > 0$$
$$T(r, t) \to T_\infty \quad \text{at} \quad r \to \infty \tag{1.4}$$

The formulation of the problem is found to be open, since the velocity of the vapor at the surface of the drop and the temperature of its surface, generally speaking, depend on the temperature distribution within the drop. However, in what follows we consider also conditions for the evaporation of a drop which are such that the characteristic times of the change in the velocity of the vapor, of the radius of the drop, and of the temperature of its surface, considerably exceed the relaxation time of the temperature in the vapor. In this case, in calculation of the temperature distribution around the drop, we can leave out of consideration the nonsteady-state character of the process, connected with a change in the quantities $a$, $T_a$, $v_a$.

If $|T_a - T_\infty| \ll T_\infty$, then in Eq. (1.3) we can leave out of consideration the dependence of $\rho$, $c_p$, and $\varkappa$ on the temperature, and can linearize this equation with respect to T.

Then

$$\frac{1}{\chi}\frac{\partial T}{\partial t} + \frac{Pa}{r^2}\frac{\partial T}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} - \frac{T - T_*}{\mu^2} \tag{1.5}$$

$$T_* = T_\infty + \frac{\varepsilon}{2}(T_a - T_\infty)\left[1 - \left(1 - \frac{a^2}{r^2}\right)^{1/2}\right]$$

$$P = \frac{v_a a}{\chi}, \quad \chi = \frac{\varkappa}{\rho c_p}, \quad \mu^2 = \frac{\varkappa}{16\alpha\sigma T_\infty^3}$$

From this it is evident that the characteristic distance at which the temperature changes is determined by the quantities $a$, $Pa$, and $\mu$, the largest of which, as was assumed in the derivation of this equation, must be negligibly small in comparison with $1/\alpha$.

It is well-known [4] that, with the radiant transfer of energy, the temperature undergoes a discontinuity on a sphere. Molecular thermal conductivity leads to a smoothing out of this discontinuity. It is shown below that convective transfer of energy also eliminates the discontinuity in the temperature.

Let the temperature vary sharply in a thin layer $r - a \sim \delta \ll a$ near the surface of a sphere.

The steady-state temperature distribution in this layer, as follows from (1.5), has the form

$$\frac{T - T_*(a)}{T_a - T_*(a)} = \exp\left(-\frac{r - a}{\delta}\right), \quad \delta = \frac{\mu^2 P}{2a}\left[1 + \left(1 + \frac{4a^2}{\mu^2 P^2}\right)^{1/2}\right]$$

Thus, in the absence of convection, $\delta = \mu$. With large Péclet numbers ($P^2 \gg 4a^2/\mu^2$), the thickness of the thermal boundary layer increases substantially $\delta = \mu^2 P/a$ and is found to be independent of the value of the coefficient of molecular thermal conductivity.

2. Molecular Thermal Conductivity and Convection. If the temperature of the medium is such that $\mu \gg a$, $\mu \gg Pa$ ($\alpha\mu \ll 1$), the temperature distribution, as is evident from Eq. (1.5), is determined by the molecular thermal conductivity and by convection. In this case, as is well-known [5], a steady-state solution can be obtained even without the assumption of a small temperature drop between the surface of the drop and infinity.

With $|T_a - T_\infty| \ll T_\infty$, the temperature distribution is described by the equation

$$\frac{1}{\chi}\frac{\partial T}{\partial t} + \frac{Pa}{r^2}\frac{\partial T}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} \tag{2.1}$$

the steady-state solution of which has the form

$$\overline{T} = \frac{1 - \exp(-Pa/r)}{1 - e^{-P}}, \quad \overline{T} = \frac{T - T_\infty}{T_a - T_\infty} \tag{2.2}$$

With large Péclet numbers, the heat flux is an exponentially small quantity. Far from the sphere ($r > Pa$), the distribution of the temperature has exactly the same form as in the absence of convection around a sphere with the effective radius $a_*$

$$\overline{T} = a_*/r, \quad a_* = Pa/(1 - e^{-P}) \tag{2.3}$$

If convective transfer of heat is neglected, then, as is well-known

$$\overline{T} = \frac{a}{r} \operatorname{erfc}\left(\frac{r-a}{2\sqrt{\chi t}}\right), \quad \operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)\, dt \tag{2.4}$$

This solution is the product of the steady-state temperature distribution, determined by the molecular thermal conductivity, by a function describing the rate of propagation of the front of the thermal wave. By analogy with (2.4), it can be assumed that the function

$$\overline{T} = \frac{1 - \exp(-Pa/r)}{1 - e^{-P}} \operatorname{erfc}\left(\frac{r-a}{2\sqrt{\chi t}}\right)$$

will differ only slightly from the exact solution of Eq. (1.4), with $\mu \gg a$, $\mu \gg Pa$.

Actually, if the function

$$\overline{T} = \Phi(r)\Psi(r, t)$$
$$\left(P\frac{a}{r^2}\frac{d\Phi}{dr} = \frac{1}{r^2}\frac{d}{dr} r^2 \frac{d\Phi}{dr}, \quad \frac{1}{\chi}\frac{\partial \Psi}{\partial t} = \frac{\partial^2 \Psi}{\partial r^2}\right)$$

is substituted into the starting equation (2.1), we can obtain

$$\Phi\left(\frac{1}{\chi}\frac{\partial \Psi}{\partial t} - \frac{\partial^2 \Psi}{\partial r^2}\right) + \Psi\left(\frac{Pa}{r^2}\frac{d\Phi}{dr} - \frac{1}{r^2}\frac{d}{dr} r^2 \frac{d\Phi}{dr}\right) = 2\left(\frac{d\Phi}{dr} + \frac{\Phi}{r}\right)\frac{\partial \Psi}{\partial r} - \frac{Pa}{r^2}\Phi\frac{\partial \Psi}{\partial r}$$

Taking into account that

$$\frac{\partial \Psi}{\partial t} \sim \frac{\Psi}{t}, \quad \frac{d\Phi}{dr} \sim \frac{Pa\Phi}{r^2}, \quad \frac{\partial \Psi}{\partial r} \sim \frac{\Psi}{\sqrt{\chi t}}$$

it is evident that the function $\Phi\Psi$ satisfies the starting equation when any of the following conditions is satisfied:

$$r^2 \gg Pa\sqrt{\chi t}, \quad r^2 \ll Pa\sqrt{\chi t}, \quad r \gg Pa$$

Thus, the relaxation time of the temperature in the region $r \sim Pa$ is a quantity on the order of $P^2a^2/\chi$.

3. Molecular Thermal Conductivity and Radiation. With small Péclet numbers ($P \ll 1$), in Eq. (1.5), convective transfer can be neglected:

$$\frac{1}{\chi}\frac{\partial rT}{\partial t} = \frac{\partial^2 rT}{\partial r^2} = -\frac{r(T - T_*)}{\mu^2} \tag{3.1}$$

The solution of this equation, with the boundary and initial conditions (1.4), has the form

$$r(T - T_\infty) = a(T_a - T_\infty)G(r - a, t) + \int_a^\infty \frac{r'}{2\mu}[T_*(r') - T_\infty][G(|r - r'|, t) - G(r + r' - 2a, t)]\, dr' \tag{3.2}$$

$$G(r, t) = \frac{1}{2}\left[\exp\left(-\frac{r}{\mu}\right)\operatorname{erfc}\left(\frac{r}{2\sqrt{\chi t}} - \frac{\sqrt{\chi t}}{\mu}\right) + \exp\left(\frac{r}{\mu}\right)\operatorname{erfc}\left(\frac{r}{2\sqrt{\chi t}} + \frac{\sqrt{\chi t}}{\mu}\right)\right] \tag{3.3}$$

The steady-state solution is described by the same formula (3.2), but with the replacement of $G(r, t)$ by the function

$$G(r) = \exp(-r/\mu)$$

As is evident from formula (3.3), in the region $r \ll \mu$, there exist two characteristic times for the change in the temperature: with small times, $r^2/\chi$, and at large times, $\mu^2/\chi$. In the region $r \gg \mu$, a steady-state temperature distribution is established after a time $t \sim \mu r/\chi$.

Let $\mu \gg a$; then, at $t > \mu^2/\chi$, with an accuracy up to terms whose order of magnitude, in accordance with formula (3.2) does not exceed $\varepsilon(a/\mu)\ln(\mu/a)$, the temperature distribution has the form

$$\bar{T} = \frac{a}{r}\exp\left(-\frac{r-a}{\mu}\right) \tag{3.4}$$

Molecular thermal conductivity is found to be considerable in a layer with a thickness $\mu$.

These results are in agreement with those obtained earlier [6, 7].

If, in the presence of convection, the condition $Pa \ll \mu$ is satisfied, then, in the region $Pa < r < \mu$, where only molecular thermal conductivity is significant, formulas (2.2) and (3.4) are applicable.

As a result of asymptotic joining, the following temperature distribution is obtained:

$$\bar{T} = \frac{1-\exp(-Pa/r)}{1-e^{-P}}\exp\left(-\frac{r-a}{\mu}\right)$$

4. Convection and Radiation. Under conditions of the predominant effect of convective-radiative transfer of energy, the nonsteady-state temperature distribution is described by the equation

$$\frac{1}{\chi}\frac{\partial T}{\partial t} + \frac{Pa}{r^2}\frac{\partial T}{\partial r} + \frac{T}{\mu^2} = \frac{T_*}{\mu^2} \tag{4.1}$$

with the initial and boundary (at $r = a$) conditions (1.4).

The solution of this equation has the form

$$\bar{T} = \eta\left(t - \frac{r^3-a^3}{3Pa\chi}\right)\exp\left(-\frac{r^3-a^3}{3\mu^2 Pa}\right) +$$

$$+ \int_a^r \frac{r'^2}{\mu^2 Pa}\frac{T_*(r') - T_\infty}{T_a - T_\infty}\eta\left(t - \frac{r^3-r'^3}{3Pa\chi}\right)\exp\left(-\frac{r^3-r'^3}{3\mu^2 Pa}\right)dr'$$

$$\eta(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

The region of applicability of this equation corresponds to the assumption that the distance at which the thermal front becomes diffuse as a result of molecular thermal conductivity is negligibly small in comparison with the distance traversed by the front after the same time, i.e.,

$$(\chi t)^{1/2} \ll (Pa\chi t)^{1/3}$$

Radiative equilibrium is established in the region $r^3 \sim 3\mu^2 Pa$ after a time $t \sim \mu^2/\chi$.

As is evident from the solution obtained, at sufficiently large times, this inequality cannot be satisfied.

The steady-state temperature distribution

$$T = T_a\exp\left(-\frac{r^3-a^3}{3\mu^2 Pa}\right) + \int_a^r \frac{r'^2 T_*(r')]}{\mu^2 Pa}\exp\left(-\frac{r^3-r'^3}{3\mu^2 Pa}\right)dr'$$

differs from that calculated in the approximation of radiative equilibrium only in the region $r^3 - a^3 \lesssim 3\mu^2 Pa$.

As an evaluation shows, at $\mu \gg a$ the transfer of energy by molecular thermal conductivity may be neglected if $\mu \ll Pa$. In this case, the transition to radiative equilibrium $r^3 \sim \mu^2 Pa$ takes place in the region $r \ll Pa$.

With $\mu \ll a$, as shown in Section 1, molecular thermal conductivity may be left out of account in Eq. (1.5) at $P^2 \gg 4a^2/\mu^2$.

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